THE LAWS OF AVANZINI

Laws of Planes moving at an angle in Air and Water

BY

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THE LAWS OF AVANZINI:

LAWS OF PLATES MOVING AT AN ANGLE IN FLUIDS

BY LT.-COL. R. DE VILLAMIL (R.E., retired)

FOREWORD

If one reflects on the great army of scientific men who are at present studying the subject of the motion of planes, at an angle, in fluids, it is very remarkable that the work of Avanzini on this subject should have escaped notice. All that appears to be known about this diligent experimentalist is in reference to what is known as "Avanzini's law," in relation to the position of the centre of pressure on a moving plate. This law is quoted by Professor Bryan and M. Alexandre Sée (to quote two only of the most eminent writers on the subject), but they omit to say from whence they get the formula. M. Alexandre Sée, in a private letter, has kindly informed me that "je crois qu'on a appelé 'Loi d'Avanzini' *une loi qu'il n'a jamais formulée lui même*, mais qu'il a étudiée." Whatever Avanzini may have said on the subject is probably to be found in the *Memorie della Academia di Padova*, but the volume containing his memoir does not appear to be in England; in any case I have not been able to find it.

Avanzini was a priest and a Professor of Mathematics in the North of Italy. The work of his to which I specially refer here was published in Bologna in the *Memorie dell' Istituto Nazionale Italiano*, in the first decade of the nineteenth century. These volumes are very rare, but by the courtesy of the Royal Society I have been enabled to give copies of the tables and plates in the following pages.

Avanzini started from the principle that there are six variables in the problem: (1) length of plate, (2) breadth of plate, (3) velocity of plate in still water, (4) angle of attack, (5) position of the centre of pressure of the fluid, and (6) the density of the fluid. In each of his series of experiments he keeps four of these constant and then studies the relation between the other two. The experiments were very carefully arranged and carried out, and the results obtained are always consistent with one another; they are therefore well worth rescuing from oblivion; even though one may not agree with all his deductions and explanations.

If we compare them with Langley's* we see how much more reliable they appear to be. For example, if we refer to the chapter on "The counterpoised eccentric plane" in the latter work we see that in some of the experiments the angle of attack varied as much as 82°, 80°, 72° and 68°; that is to say 41°, 40°, 36° and 34° on either side of the *mean* angle. Under these circumstances it is impossible to attach any very great importance to the exact value of the "mean angle"; more especially as, when the axis of the plate is fixed centrally, the angle of attack was found to vary from 78° to 90°. Langley, referring to his Table XVII. on this subject, says:—"It will be noticed that this angle $(90° - \theta)$ is 5°·5 for the case when the axis of rotation passes through the centre of the plane—a setting for which the plane must be vertical. This observed angle of 5°·5 is to be explained, not by a tipping of the plane, but by a tipping of the line of reference due to a yielding of the supports, &c., to the wind of rotation. This angular deflection, therefore, becomes a correction to be applied to all the observations." This would appear to be a very rough and ready method of employing an "index error" *which, presumably, varies with the velocity*. Avanzini tells us his plates did not oscillate, visibly, and that the differences in the angles observed did not exceed a few minutes of a degree (*pochi minuti*); there was also no index error. In Langley's experiments the oscillations of the plate appear to have been so great as to completely mask Avanzini's second law—that is that

$$\theta \propto \frac{1}{f(V)}.$$

where θ = angle of attack, and V = velocity of the plate.

In studying the subject it is, perhaps, always best to start with experiments in water; they are more reliable, being subject to fewer disturbances. The experiments here quoted were, however, made in both fluids.

* Experiments in Aerodynamics, 1891.

The velocities employed by Avanzini were rather low; and it may be contended that these laws *may not be equally true for higher speeds.* Whether this is so, or not, is for experiment to decide; but there is no apparent reason why there should be a change of law at higher velocities (*which are not excessive*).

As regards the measures of length, some trouble has been experienced in trying to find out what is the size of the *piede* (which has been translated as *foot*). Colonel Duchemin, writing about Avanzini's work, says he employed the "Turin foot," which he says was = $0_{m}.5136524$, and was divided into 12 inches (*pollici*); 1 inch = 12 lines (*linee*), 1 line = 12 points. The Turin foot is sometimes called the "Piedmont foot" or the "Sardinian foot" (*Piede liprando*), and is stated in tables to be = 20·22 English inches.

If Avanzini employed this *piede* (and there is reason to believe it was the "scientific measure" in the North of Italy at this time) then the velocities were more than 60 per cent. greater than they would appear to be at first sight. The exact value is of no special importance here as the laws enunciated, are qualitative only.

Law 1

WHEN a rectangular lamina is immersed in a liquid, at rest (except when its surface is horizontal) the centre of pressure of the liquid on the plate will fall between the centre of figure and the lower edge of the plate: this is a natural consequence of the pressure increasing with the depth of immersion.

When, however, the lamina moves in the liquid, all this is changed. If the angle of attack is a right angle, the centre of pressure will be found to be at the centre of figure of the plate; whilst if the angle of attack is acute the centre of pressure will be *between the centre of figure and the leading edge*—whether the leading edge be horizontal and above, or below, the trailing edge, or whether the leading edge be vertical.

There is no reason for believing that Newton was aware of this fact; for, even after his time, that great quartet, Euler, Bernouilli, S'Gravesande and d'Alembert, made no mention of it. The last of these even says "it must be observed that the centre of pressure of the fluid is, or must be assumed to be at the centre of gravity of the figure.†

Even at the close of the eighteenth century (1798) neither Vince, nor that very close observer Dubuat, make any reference to it. ‡

Very early in the nineteenth century Avanzini carried out a series of remarkable experiments, from which he deduced the laws of the aeroplane. His paper was presented to the *Istituto Nazionale Italiano* on the 2nd July, 1804, and was published in the first part of the first volume of the Memoirs in 1806. These laws I have, therefore, called the "Laws of Avanzini," as he appears to be the first to have pointed them out; and to prevent any misunderstanding I define a law of Nature as an invariable sequence. To give an example: if a stone is dropped it *invariably* falls to the ground; this one may call the *qualitative* law of gravitation. Avanzini's laws, which are very imperfectly known, even at the present day, are only *qualitative* laws; the *quantitative* laws are still very obscure. I might add, further, that even the name of Avanzini is hardly remembered.

Description of Apparatus

The apparatus employed in carrying out the experiments is very fully shown in the accompanying Plates I. and II., which are photographic reproductions of the plates in the Memoirs of the *Istituto Nazionale Italiano.*

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† Il faut observer que le centre de pression du Fluide est ou doit être censé au centre de gravité de la Figure. (*Essai d'une nouvelle Théorie de la résistance des fluides.*)

‡ It appears doubtful if, in 1889—nearly another century later—Lilienthal was not ignorant of this; though Colonel Duchemin in 1842 had gone into the question very fully.

Fig. 1, Table II., is a general view of the railway, showing the carriage with an experimental plate held in position. The track was about 140 feet long and the carriage was moved along it by means of falling weights, as shown in Fig. 1, an arrangement which needs no explanation. To allow for the diminished resistance of the cord, as it was wound up, part of the accelerating weights were chains, Fig. 12, which, by resting on the ground, gradually reduced the *effective* weights ; a very uniform motion of the carriage was thus obtained. Sights were fixed at every ten of the last seventy feet of the run, OO, Fig. 1, and trained observers recorded the time taken by the plate in moving past each ten feet. The motion was found to be perfectly uniform ; and two heavy balls suspended by cords, Q Q, Fig. 1, acted as buffers in checking the motion of the carriage at the end of the run.

Fig. 2 gives an enlarged view of the carriage, which had brass wheels and side rollers, as shown. Figs. 3 and 4 are still more enlarged views of the Y-shaped plate holder, showing how it was fixed to the carriage. It will be seen that the leading edge of the plate could be fixed horizontally, vertically, or at any required angle. Between the forks of the Y-piece, the small plate with semi-circular ears (Fig. 6) was placed and held in position by the small screws W W, Fig. 4. This plate is shown in position in Fig. 9.

Since the plate was free to turn about the pivots and it was necessary to measure the angle at which it was moving, the arrangement K K, Fig. 1—shown in detail in Fig. 14—was adopted. The sharp point O could be adjusted as desired, so that the plate should strike it and be marked by it. The measurement of the angle was then made as shown in Figs. 3 and 11, Table I. ; where angles are shown as being measured both vertically and horizontally.

Fig. 15, Table I, is a representation of the brick canal (150 feet long) which could be filled, or not, with water ; and in which the plate moved. It will be evident that this arrangement was very superior to any system of whirling table, since all centrifugal action was absent, and there were not any eddies produced by the apparatus itself. Also when the plates were moving in the air, they were sheltered from any disturbing winds.

Since it was necessary to be quite sure that there was no disturbance of the fluid caused by " boundaries "—in order that the fluid might be considered to be of " indefinite " extent—Avanzini had small pith balls, fastened by short pieces of fine silk, on the inside of the canal, for the experiments in water. In all cases, whatever the velocity or the angle of inclination of the plates, it was found that no movement of these pith balls could be observed. Similarly, when the canal was empty, he strewed very light feathers on the bottom ; the moving plates causing no visible disturbance of these. It was evident, therefore, that there were no perceptible eddies in the fluids resulting from the boundaries, and that the fluids might consequently be considered as being " indefinite."

All being ready the experimental plate was fixed to the small plate, Fig. 6 (Tav. II.), and the carriage was released.

During the first twenty feet or so, the plate oscillated ; but when the motion of the carriage became uniform the plate came to rest at a fixed angle and remained at this angle during the travel of considerably more than the last seventy feet. The angle was then measured.

As Avanzini says :—

" 1°. To cause the lamina to move normally to the direction of its motion, it is necessary that the axis of the pivots, or of its own equilibrium, should pass through the centre of figure ; 2°, to move it at a given angle to the direction of its own motion it is necessary that the aforesaid axis of equilibrium should be between the leading edge of the lamina and the centre of figure, and at a greater or less distance from this centre according to the smaller or greater inclination that it is desired the lamina should assume." It was to prove this that the first series of experiments was carried out.

The sizes and weights of the plates experimented with are given in the following table:—

Water.

Lamina.	Length.	Breadth.	Weight.
I.	9 inches	6 inches	16 ounces
II.	9 "	4 "	15 "
III.	9 "	3 "	13 "
IV.	6 "	4 "	7 "
V.	6 "	3 "	6 "
VI.	6 "	2 "	4 "

Air.

Lamina.	Length.	Breadth.	Weight.
I.	18 inches	4 inches	5 ounces
II.	13 in. 6 lines	4 "	4 "
III.	9 inches	4 "	$2\frac{1}{2}$ "
IV.	9 "	6 "	4 "
V.	9 "	8 "	5 "

12 lines = 1 inch; 12 inches = 1 foot (*piede*).

Weights are in Italian ounces.

If the foot was the *piede liprando*, of over 20 English inches in length, it is evident that the "inches" (*pollici*) were about 1·6 English inches in length, the "lines" being one-twelfth of this.

For the experiments in water the plates were of iron, smooth and flat and of the thickness of about half a line—one-twentyfourth of an inch, since 12 lines = one inch. For the experiments in air the plates were of wood about one line (one-twelfth of an inch) in thickness.

In the following tables the "Distance of the axis of equilibrium" from the "centre of figure" of the plate is always expressed in *twentyfourths of the length of the lamina—or twelfths of the half length.* The "angle" of the plate is the angle, on a vertical section (if the upper edge of the plate is horizontal) between the face of the plate and the line of motion; this is what the French call the "angle of attack."

Avanzini says he "thinks it important to state:—1°. that each of these experiments is the mean result of twenty or more which were made; 2°. that the maximum difference between these latter, in the measurement of the angles, did not exceed a few minutes of a degree."

In order to balance the plates when the axis of support did not pass through the centre of figure, it was necessary to cause the centre of gravity to pass through this axis. To do this small holes were bored in the plates, and these were filled with small heavy bodies in the case of the wooden plates, and with light bodies in the case of the iron plates. By this means a proper balance was obtained and the action of gravity was eliminated; the plates could then, when moving steadily, be considered as having no *weight*.

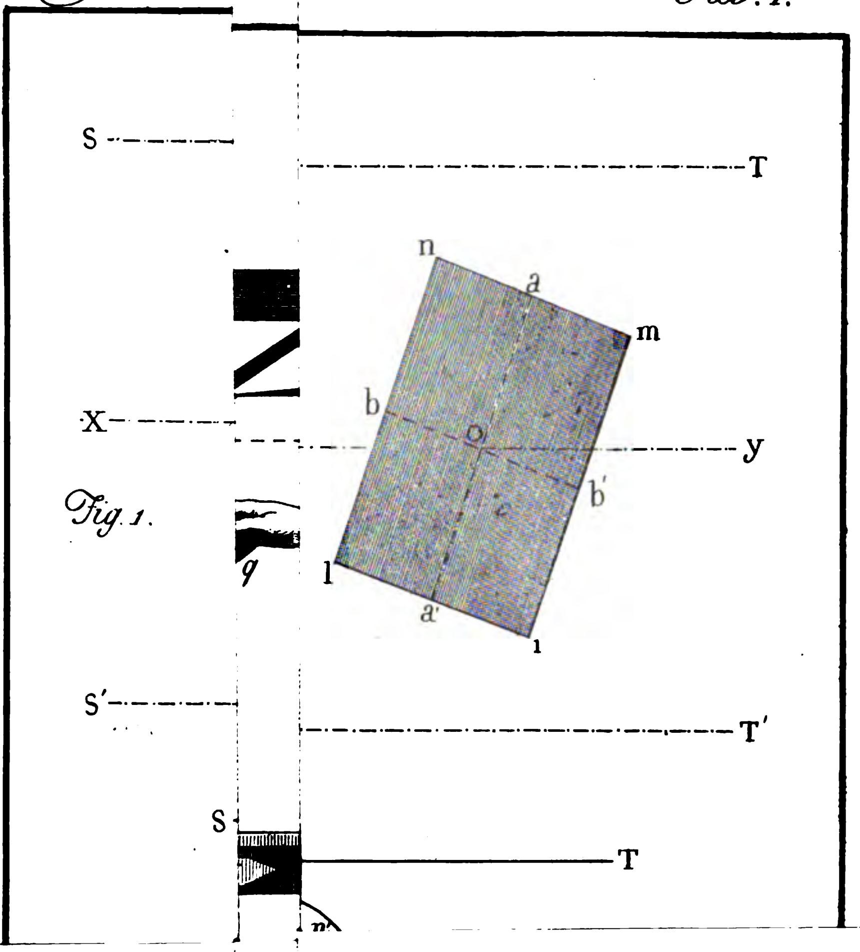
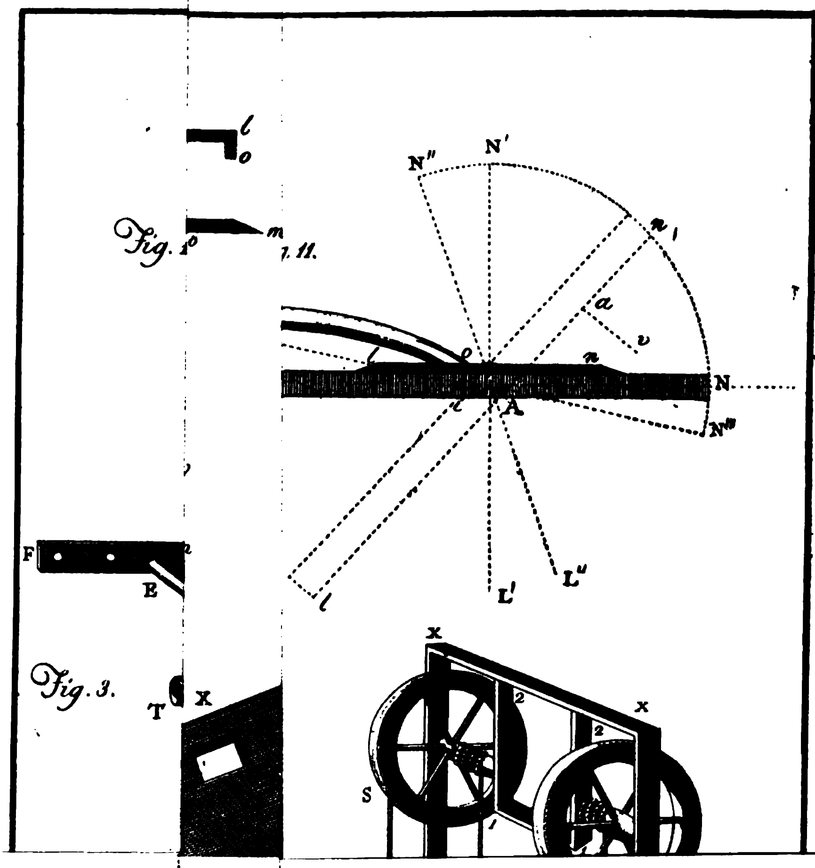


Fig. 1.

Pag. 33. Tav. II



In the problem to be solved there are six variables :—

1. Length of plate.
2. Breadth of plate.
3. Velocity of plate.
4. Angle of inclination of plate (angle of attack).
5. Position of axis of equilibrium when the motion is "steady."
6. Density of the fluid.

By keeping any four of these constant it is easy to see how the remaining two depend on one another. It will also be obvious that if the plate, which is free to turn about the axis of suspension, moves steadily at a constant angle, the centre of pressure must pass through this axis of suspension ; if it were situated anywhere else a couple would be formed and the plate would rotate about the axis.

The first series of experiments, given below, were carried out to show where the axis of equilibrium was situated.

Position of axis of equilibrium

In the following experiments attention was specially directed towards the relation between the "angle of attack" and the position of the axis of equilibrium. The first fourteen series were conducted in water and the remaining ten in air. Viewed generally, the results were the same in both cases.

EXPERIMENTS CONDUCTED IN WATER

Experiment I

Lamina I. Velocity = 1 ft. in 0·60″

Dist. of axis ..	0	1	2	3	4
Angle	90°	53°	34°	25° 40′	21° 30′

Experiment II

Lamina I. Velocity = 1 ft. in 0·30″

Dist. of axis	0	3	4
Angle	90°	15° 40′	12° 25′

Experiment III

Lamina II. Velocity = 1 ft. in 0·60″

Dist. of axis	0	3	4
Angle	90°	25°	21°

Experiment IV

Lamina II. Velocity = 1 ft. in 0·47″

Dist. of axis	0	4	
Angle	90°	18° 30′	

Experiment V

Lamina II. Velocity = 1 ft. in 0·30″

Dist. of axis	0	3	4
Angle	90°	13° 20′	12°

Experiment VI

Lamina II. Velocity = 1 ft. in 0·23″

Dist. of axis	0	4	
Angle	90°	10° 45′	

Experiment VII

Lamina III. Velocity = 1 ft. in 0·60″

Distance of the axis of equilibrium	0	3	4
Angle	90°	24° 15′	20° 25′

Experiment VIII

Lamina III. Velocity = 1 ft. in 0·47″

Dist of axis	0	4	
Angle	90°	17° 35′	

Experiment IX

Lamina IV. Velocity = 1 ft. in 0·60″

Dist. of axis	0	1	2	3	4
Angle	90°	54°	34° 30′	25° 30′	21° 30′

Experiment X

Lamina IV. Velocity = 1 ft. in 0·38″

Dist. of axis	0	4	
Angle	90°	15°	

Experiment XI

Lamina IV. Velocity = 1 ft. in 0·19″

Dist. of axis	0	4	
Angle	90°	8° 44′	

Experiment XII

Lamina V. Velocity = 1 ft. in 0·60″

Dist. of axis	0	2	
Angle	90°	34°	

Experiment XIII

Lamina VI. Velocity = 1 ft. in 0·60″

Dist. of axis	0	2	
Angle	90°	33° 25′	

Experiment XIV

Lamina VI. Velocity = 1 ft. in 0·38″

Dist. of axis	0	2	
Angle	90°	27° 50′	

EXPERIMENTS CONDUCTED IN AIR

Experiment I

Lamina I. Velocity = 1 ft. in 0·15″

Dist. of axis	0	4	5
Angle	90°	13° 56′	6° 37′

Experiment II

Lamina I. Velocity = 1 ft. in 0·30″

Dist. of axis	0	4	5
Angle	90°	15° 33′	9° 25′

Experiment III

Lamina II. Velocity = 1 ft. in 0·15″

Dist. of axis ..	0	3	4	5
Angle	90°	37° 30′	19° 15′	7° 46′

Experiment IV

Lamina II. Velocity = 1 ft. in 0·30″

Dist. of axis	0	4	5
Angle	90°	20° 38′	9° 30′

Experiment V

Lamina III. Velocity = 1 ft. in 0·15″

Dist. of axis	0	4	5
Angle	90°	26° 6′	12° 26′

Experiment VI

Lamina III. Velocity = 1 ft. in 0·30″

Dist. of axis ..	0	3	4	5
Angle	90°	52° 22′	27° 41′	15° 20′

Experiment VII

Lamina IV. Velocity = 1 ft. in 0·15″

Dist. of axis	0	4	5
Angle	90°	27° 39′	15° 36′

Experiment VIII

Lamina IV. Velocity = 1 ft. in 0·30″

Dist. of axis ..	0	3	4	5
Velocity	90°	49° 50′	28° 30′	17° 50′

Experiment IX

Lamina V. Velocity = 1 ft. in 0·15″

Dist. of axis	0	4	5
Angle	90°	28° 9′	18° 40′

Experiment X

Lamina V. Velocity = 1 ft. in 0·30″

Dist. of axis	0	4	5
Angle	90°	29° 15′	20° 35′

" From the foregoing tables it appears evident that in both of the fluids, and at all velocities and for all sizes of the plate, the angle under which it comes to rest is always a right angle when the axis passes through the centre of figure ; and always acute when this axis passes between the centre of figure and the superior edge of the lamina, becoming acuter as the distance of the axis from the centre of figure increases."

" The differences between the *results of the experiments* on the plates of the indicated dimensions and the *results deduced from theory* are, as will be seen, *so great* that . . . I have thought it useless to renew the experiments with lamina of greater size."

The words italicised show that the laws deduced from the experiments were not known at the time Avanzini wrote this.

Avanzini's First Law

When the " angle of attack " of a small rectangular lamina, moving steadily in a liquid, is a right angle, the centre of pressure passes through the centre of figure ; when, however, the angle of attack is acute, this centre of pressure is situated between the centre of figure and the leading edge, and approaches this leading edge as the angle of attack becomes acuter. This is equally true for both water and air.

Laws 2 and 3

Although it would not be very difficult to trace the relations between the other variables of the problem, from the foregoing experiments, nevertheless Avanzini thought it " would not be superfluous to represent in several tables these same variations."

The next series of experiments were conducted with a view to showing the relation between the velocity of the plate and the angle of attack. The apparatus was the same and the results are given in the following tables ;—

ANGLES CORRESPONDING TO VARIOUS VELOCITIES

IN WATER

EXPERIMENT I

Lamina II. Dist. of axis of equilibrium = 4

Velocity	1 ft. in 0·30″	1 ft. in 0·47″	1 ft. in 0·60″
Angle	12°	18° 30′	21°

EXPERIMENT II

Lamina IV. Dist. of axis = 4

Velocity	1 ft. in 0·38″	1 ft. in 0·60″	
Angle	15°	21° 30′	

EXPERIMENT III

Lamina I. Dist. of axis = 4.

Velocity	1 ft. in 0·30″	1 ft. in 0·60″	
Angle	12° 25′	21° 30′	

EXPERIMENT IV

Lamina II. Dist. of axis = 4.

Velocity	1 ft. in 0·30″	1 ft. in 0·60″	
Angle	13° 20′	25°	

EXPERIMENT V

Lamina III. Dist. of axis = 3.

Velocity	1 ft. in 0·47″	1 ft. in 0·60″	
Angle	17° 35′	20° 25′	

EXPERIMENTS CONDUCTED IN AIR

EXPERIMENT I

Lamina I. Dist. of axis = 4.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	13° 56′	15° 33″ (*sic*)	

Experiment II

Lamina I. Dist. of axis = 5.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	6° 37′	9° 25′	

Experiment III

Lamina II. Dist. of axis = 4.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	19° 15′	20° 38′	

Experiment IV

Lamina II. Dist. of axis = 5.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	7° 46′	9° 30″ (*sic*)	

Experiment V

Lamina III. Dist. of axis = 4.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	26° 6′	27° 41′	

Experiment VI

Lamina III. Dist. of axis = 5.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	12° 26′	15° 20′	

Experiment VII

Lamina IV. Dist. of axis = 4.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	27° 39′	28° 30′	

Experiment VIII

Lamina IV. Dist. of axis = 5.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	15° 36′	17° 50′	

Experiment IX

Lamina V. Dist. of axis = 4.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	28° 9′	29° 15′	

Experiment X

Lamina V. Dist. of axis = 5.

Velocity	1 ft. in 0·15″	1 ft. in 0·30″	
Angle	18° 40′	20° 35′	

In reference to these Avanzini says :—" From the preceding tables it becomes manifest that the acute angle, at which a given lamina comes to rest, whatever may be its size or the distance of the axis of equilibrium, and the fluid in which it is moving, becomes smaller as the velocity of the lamina increases."

" From that follows the necessary consequence that the centre of resistance encountered by a thin and smooth rectangular body moving in water, or in air, at rest, at any angle acute to the direction of its motion, will be *less distant from the centre of figure* of the anterior surface of the solid, *as the velocity increases*."*—(Avanzini.)

I have been careful to give Avanzini's own words because I disagree with his " necessary consequence " ; and I have italicised the words to which I wish to draw particular attention.

If I read his meaning correctly, it is that *as the velocity increases* the centre of pressure on the *leading surface* of the plate *shifts* gradually *towards the centre of figure* and so tends to cause the plate to rotate about the axis of support and by so doing *reduces the angle of attack.*

The explanation appears plausible since, (1) the plate undoubtedly rotates about this axis of support, and (2) there must be a couple produced to cause this rotation.

We know, however, by Avanzini's first law, that the centre of pressure *approaches the leading edge of the lamina as the angle of attack becomes acuter.* We should therefore have a centre of pressure which is getting " less distant from the centre of figure," whilst it is also " approaching the leading edge, as the angle of attack becomes more acute "—a contradiction which it is impossible to accept. The explanation must be sought for elsewhere.

* Da ciò segue di necessaria consequenza che il centro di resistenza incontrata da un piano e sottil corpo rettangolare moventesi per l'acqua o per l'aria tranquilla sotto un qualunque angolo acuto colla direzione del suo movimento, sarà *tanto meno distante dal centro di grandezza della superficie anteriore* del solido, quanto *piu grande* sarà *la sua velocità*,

The pressure on the plate may be considered as being divided into two parts: the pressure on the anterior surface and that on the posterior surface; that on the latter being less than that on the former. For simplicity I will call the pressure on the anterior surface *positive* and that on the posterior surface *negative*. Now, referring to Fig. 6, Table I., let us suppose the plate l^1n^1 to be moving steadily at a fixed angle and that o is the centre of figure, whilst o^1 is the axis of equilibrium. It is manifest that, since there is equilibrium, the centre of pressure of the *positive* pressure must pass through the point o^1; also the centre of pressure of the *negative* pressure must equally pass through o^1. The moment of the pressure on o^1l^1 = the moment of the pressure on o^1n^1. Since the plate is completely immersed in the fluid, the point of greatest *positive* pressure must be at o^1, and therefore one may call the point o^1 the "Divide"—the point from which the fluid flows upwards and downwards past the edges of the plate; more fluid will, of course, flow past l^1 than past n^1. Similarly, the point of least *negative* pressure—measured negatively, of course—must also be at o^1.

If the velocity of the lamina be increased there appears to be no reason why the position of the "Divide" should be altered—all the reasons would appear to point to the reverse. There are, however, very strong reasons for believing that the centre of pressure of the *negative* pressure will be shifted from o^1 towards n^1. This by reducing the pressure—*measured positively*—on the posterior surface of l^1o^1 will produce a *couple* which will cause the plate to rotate, clockwise, in Fig. 6. In other words, the centre of pressure (measured positively) on the posterior surface will be shifted from o^1 towards the leading edge, n^1, whilst the position of the "Divide" will not change. When the plate has revolved through a small angle and the *proper* "angle of attack" has been reached, the centre of *negative* pressure will once more return to o^1 and remain there. In other words, the centre of the ring vortex formed behind the plate will oscillate gently from o^1 towards n^1 and eventually back again to o^1.

Whether this explanation be considered satisfactory or not, I prefer to state Avanzini's second law without his "necessary consequence."

Avanzini's Second Law

When the axis of equilibrium is at a fixed distance from the centre of figure, the velocity and the angle of attack alone varying, as the velocity *increases* the "angle of attack" *decreases*.

In other words, the angle of attack varies as an inverse function of the velocity, or, in algebraical shorthand:—

$$\theta \backsim f\left(\frac{1}{V}\right)$$

where θ = angle of attack, and
V = velocity.

The next series of experiments were carried out to show the relation that exists between the length of the plate and the angle of attack.

ANGLES CORRESPONDING TO VARIOUS LENGTHS

WATER

EXPERIMENT I

Breadth of Lamina = 4 ins. Dist. of axis = 3.

Velocity = 1 ft. in 0·60″

Length	9 inches	6 inches	
Angle	25°	25° 30′	

Experiment II

Breadth of Lamina = 4 ins. Dist. of axis = 4.
Velocity = 1 ft. in 0·47″

Length	9 inches	6 inches	
Angle	18° 30′	19°	

Experiment III

Breadth of Lamina = 4 ins. Dist. of axis = 4.
Velocity = 1 ft. in 0·19″

Length	9 inches	6 inches	
Angle	8°	8° 4′	

ANGLES CORRESPONDING TO VARIOUS LENGTHS

Air

Experiment I

Breadth of Lamina = 4 ins. Dist. of axis = 4.
Velocity = 1 ft. in 0·15″

Length	18 inches	13½ inches	9 inches
Angle	13° 56′	19° 15′	26° 6′

Experiment II

Breadth of Lamina = 4 ins. Dist. of axis = 4.
Velocity = 1 ft. in 0·30″

Length	18 inches	13½ inches	9 inches
Angle	15° 33′	20° 38′	27° 4′

Experiment III

Breadth of Lamina = 4 ins. Dist. of axis = 5.
Velocity = 1 ft. in. 0·15″

Length	18 inches	13½ inches	9 inches
Angle	6° 37′	7° 46′	12° 26′

EXPERIMENT IV

Breadth of Lamina = 4 ins. Dist. of axis = 5.

Velocity = 1 ft. in 0·30″

Length	18 inches	13½ inches	9 inches
Angle	9° 25′	9° 30′	15° 20′

"These tables prove that augmenting the length of the plates decreases the angle under which they remain inclined whilst travelling the seventy feet." To this is unfortunately added :—"By the reasoning in the last section they show that the centre of resistance encountered by a thin and smooth rectangular body moving in water, or air, at rest under any acute angle will be *less distant from the centre of figure of the anterior surface of the solid*, as the longitudinal side of the same surface becomes greater."*

The argument is unsound. There is one position, and one position only, for the centre of pressure when the plate is moving without rotation, and that is through the axis of equilibrium.†

Avanzini's Third Law

If the length of the plate be increased, the breadth, velocity and distance of the axis from the centre of figure being kept constant ; as the length of the plate increases the angle of attack decreases ; or—

$$\theta \backsim f\left(\frac{1}{L}\right)$$

where L = length of plate and θ = angle of attack. This is most marked at high velocities.

Laws 4 and 5

In the next series of experiments the length of the plate, velocity and distance of the axis of equilibrium were kept constant, the only variables being the breadth of the plate and the angle of attack.

ANGLES CORRESPONDING TO VARIOUS BREADTHS

WATER

EXPERIMENT I

Length of Lamina, 9 ins. Velocity = 1 ft. in 0·60″

Distance of the axis of equilibrium = 3.

Breadth	6 inches	4 inches	3 inches
Angle	25° 40′	25°	24° 15′

* Poi ragionamenti del § precedente ci manifesta che il centro di resistenza incontrata da un piano e sottil corpo rettangolare moventesi per l'acqua, e per l'aria tranquilla sotto un qualunque angolo acuto sarà tanto meno distante dal centro di grandezza della superficie anteriore del solido, quanto più sarà grande il lato longitudinale della superficie medesima.

† From what was said about the second law, the reader will easily see that the correct explanation is that *increasing the length of the plate* produces the *same effect on the vortex at the rear of the lamina*, as increasing the velocity. There appears to be no reason for supposing that the position of the "divide" is altered.

Experiment II

Length of Lamina = 9 ins. Velocity = 1 ft. in 0·60″
Distance of axis of equilibrium = 4.

Breadth	6 inches	4 inches	3 inches
Angle	21° 30′	21°	20° 25′

Experiment III

Length of Lamina = 6 ins. Velocity = 1 ft. in 0·60″
Distance of axis of equilibrium = 2.

Breadth	4 inches	3 inches	2 inches
Angle	34° 30′	34°	33° 25′

Air

Experiment I

Length of Lamina = 9 ins. Velocity = 1 ft. in 0·15″
Distance of the axis of equilibrium = 4.

Breadth	8 inches	6 inches	4 inches
Angle	28°	27° 39′	26° 6′

Experiment II

Length of Lamina = 9 ins. Velocity = 1 ft. in 0·30″
Distance of axis of equilibrium = 4.

Breadth	8 inches	6 inches	4 inches
Angle	29° 15′	28° 30′	27° 41′

Experiment III

Length of Lamina = 9 ins. Velocity = 1 ft. in 0·15″
Distance of the axis of equilibrium = 5.

Breadth	8 inches	6 inches	4 inches
Angle	18° 40′	15° 36′	12° 26′

Experiment IV

Length of Lamina = 9 ins. Velocity = 1 ft. in 0·30″

Distance of axis of equilibrium = 5.

Breadth	8 inches	6 inches	4 inches
Angle	20° 35′	17° 50′	15° 20′

"From the preceding tables it appears that increasing the breadth of the plates sensibly increases the acute angle under which they come to rest." After this follows a remark that this could only happen by the centre of pressure moving away from the centre of figure—an argument which appears unsound for the reasons previously given.*

Avanzini's Fourth Law

If the distance of the axis of suspension of the plate be kept at a fixed distance from the centre of figure; and if the length and velocity of the plate be kept constant the angle of attack will be *sensibly* increased by increasing the breadth of the plate; or—

$$\theta \propto f(b)$$

where b = breadth of plate and θ = angle of attack.

It will be observed that increasing the breadth of the plate acts in the opposite manner to increasing the length. This shows, what has been found out practically by aeroplane builders, the advantage of making the attack *in line* rather than in *column* formation.

Since increasing the breadth of the plate tends to *increase* the angle of attack *very slightly*, whilst increasing the length of the plate *diminishes* this angle of attack in a *much greater degree*, it would follow that there must be some proportion of the length to the breadth when these two effects will neutralise one another. This proportion, where the breadth much exceeds the depth, would apparently be the best *theoretical* shape for an aeroplane wing where stability is sought for. Doubtless, the aeroplane builders have, by a process of trial and error, *practically* found this proportion; nevertheless, it is very necessary, in order to co-ordinate one's ideas on the subject, to have a *theoretical* explanation of the reasons for this proportion.

The last series of experiments were conducted to show the effect of the density of the fluid on the angle of attack. Length, breadth and velocity of the lamina were kept constant; as well as the distance of the axis of equilibrium; the experiments were then made in air and water, with the following results:—

ANGLES CORRESPONDING TO THE TWO KINDS OF FLUIDS

Experiment I

Length of Lamina = 9 ins. Breadth = 4 ins.
Velocity = 1 ft. in 0·30″ Dist. of axis = 4.

Angle in water .. .. 12°
Angle in air .. .. 27° 41′

* It will be evident that, in this case, increasing the breadth of the plate tends, *sensibly* (though slightly) to shift the centre of the vortex behind the lamina, from o^1 towards i^1; after which it again returns to o^1. The rotation, in this case, being *counter-clockwise* in Fig. 6.

Experiment II

Length of Lamina = 9 ins. Breadth = 6 ins.
Velocity = 1 ft. in 0·30″ Dist. of axis = 4.

Angle in water	12° 25′
Angle in air	28° 30′

Experiment III

Length of Lamina = 9 ins. Breadth = 6 ins.
Velocity = 1 ft. in 0·30″ Dist. of axis = 3.

Angle in water	25° 40′
Angle in air	49° 50′

Experiment IV

Length of Lamina = 9 ins. Breadth = 4 ins.
Velocity = 1 ft. in 0·30″ Dist. of axis = 3.

Angle of water	13° 20′
Angle of air	52° 22′

" These tables show that the angles under which the plates come to rest were more acute when they were moving in water than when they were travelling in air."

To this remark is added :—" This evidently proves, by the previous reasoning, that the centre of resistance of water moves less than the centre of resistance of the air from the centre of figure of the lamina." This is a *non-sequitur*, the centre of resistance in both cases passing through the axis of equilibrium.

Avanzini's Fifth Law

Length, breadth and velocity of lamina being constant, as well as the distance of the axis of equilibrium, the angle of attack varies inversely as some function of the density of the fluid ; or—

$$\theta \backsim f\left(\frac{1}{\rho}\right)$$

where ρ = density and θ = angle of attack.

From these five laws we may deduce, generally, that—

$$\theta \backsim \frac{f\,(b)}{f\,(\rho) \times f\,(L) \times f\,(V)}$$

It is for experiment to determine what form these *functions* take. Up to the present there is not sufficient information to say what they are ; Avanzini's experiments only give 2 or 3 (very rarely 4) points on the curves, which are quite insufficient data. If these experiments were repeated, with the very superior appliances for recording results now available, much very valuable information would be obtained. Really useful curves could be drawn and the correct forms of the functions obtained. When this has been done it will be possible to give the *quantitative* laws of the aeroplane in probably as simple form as those *qualitative* ones of Avanzini which have just been explained. All whirling table experiments are open to suspicion, for many obvious reasons.

A careful study of these laws would prevent anyone accepting certain formulæ which have been proposed for finding the position of the centre of pressure on a rectangular plate moving at an acute angle in a fluid. For example, M. Alexandre Sée (" Les lois expérimentales de l'aviation ") quotes Avanzini as having proposed the formula,

$$\frac{y}{2h} = 0{\cdot}3\ (1 - \sin i)$$

where y = distance of C. of P. from C. of F.
h = half-length of the plate.
i = angle of attack.

This formula violates Law II.: for since the angle of attack varies *inversely* as some function of the velocity, *when the distance between the centre of pressure and the centre of figure is constant*, it follows that the *position of the centre of pressure* might be the same *for a variety of angles of attack.**

Jöessel (Alexandre Sée, " Lois expérimentales," &c.) proposed a slight variation on this, viz. :—

$$\frac{y}{2h} = 0{\cdot}305\ (1 - \sin i)$$

which is open to the same objection.

Both these formulæ, further, violate Law V., where angle of attack varies *inversely* as some function of the *density* of the fluid, *when the position of the centre of pressure remains constant.* Equally the *breadth* of the plate does not enter into the formula ; and this, though no doubt not *very important*, should be taken into account..

There are strong reasons for believing that the formula is more complicated than either of these, and (2) that y becomes a *maximum*, afterwards decreasing a little.

To assist in understanding the subject it is advisable to study the diagrams of the vortices given in Figs. 5 and 6, Tav. I., of Avanzini's *Memoria.* These were drawn from theoretical deductions and were afterwards found to be correct from experimental observations, made by means of small threads of silk, as well as by the use of small balls of the same density as the liquid and which moved past the plate. Referring to Fig. 5, which shows the vortex formed when the angle of attack is a right angle, this form of vortex is not stable when the motion of the plate is being accelerated—which is, of course, the case when the motion is kept *uniformly steady* in the experiment. Professor Osborne Reynolds has shown, however, that if the plate be given a *sharp push* and *then released*, this vortex is formed and *remains stable.* It appears to be purely a question of allowing the vortex to move along *at its own pace.* In the ordinary way, when the plate is being accelerated, the vortex falls to the rear a little ; another vortex appears to be formed in front of it, and the first vortex is destroyed. These broken vortices then form what are called " eddies." This vortex is of the same type as those formed when " smoke-rings " are made—a form of spherical vortex. I have dealt, at some considerable length, on the formation of these vortices as well as of smoke-rings in the " A.B.C. of Hydrodynamics," showing how they are formed as well as how they moved. The explanation of the formation of smoke-rings was, however, purely theoretical, for I quite failed in all my attempts at photographing them *whilst being formed.* In the September and October numbers of the *Journal of the Franklin Institute* (1911), I see that Mr. Edwin F. Northrup has successfully accomplished this. He gives some very beautiful photographs of rings (formed in water) at the very earliest stages of formation, and these agree remarkably closely with theory, as well as with Avanzini's Fig. 5.

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* Whether Avanzini ever proposed this formula, or not, I am unable to say. I have never seen the paper, nor have I ever come across anybody who had ; but in any case it does not agree with the results of the experiments which he undoubtedly published and which I have seen and studied.

In Fig. 6 we see the form of the vortex when the angle of attack is acute—all that has been said about the vortex in Fig. 5 applies equally to this one; but this latter has some special peculiarities of its own. The centre of the vortex is not opposite the centre of figure—the vortex is not symmetrical. The strength of that part of the vortex which is nearest to n^1 is not equal to that of the part near l^1. If the vortex were free—as it would be in the case of a "smoke-ring" vortex—the half nearest l^1 would travel faster than the part near n^1, and it would eventually be *in front* of this half, instead of in rear. The half near n^1 would thus be the stronger and would overtake the then leading half. The vortex would consequently oscillate as smoke-rings are well known to do. The centre of pressure on the posterior side of the plate would therefore shift about and oscillate through the centre of figure of the lamina. Increasing the velocity of the moving plate shifts this centre of pressure towards n^1 thus reducing the pressure on l^1o^1 and so causing the angle of attack to be reduced, as stated in Law II.

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APPENDIX

IT may be interesting to the reader to know that Avanzini carried out a series of experiments with plates *partly* immersed in water. The apparatus was the same; and so needs no further description. The results were presented to the *Istituto Nazionale Italiano* on the 11th September, 1807, and his paper was published in the first part of the second volume of the *Memorie*. The results are very similar to those found with plates completely immersed: the centre of pressure being found to be invariably between the centre of figure (of the immersed part of the plate) and the surface of the liquid. The centre of pressure was, also, further away from the "centre of figure" as the "angle of attack" decreased.

There is one point which, perhaps, calls for special notice, and which, I think, led this savant into error. He found that when the angle of attack was 90°—when the surface of the plate was normal to the direction of motion—the centre of pressure was invariably *above* the centre of figure, and not at the centre of figure, as in the previous experiments. Fig. 1, which is a photographic copy from Avanzini's plate, will explain what I mean. o is the centre of figure, whilst e is the centre of pressure, as found *experimentally*. The vortex, shown behind the plate, was worked out theoretically and then found to be correct by experiment. The error that, I think, Avanzini has fallen into is that he has drawn the stream-lines as dividing at e Y—whereas the "divide" should pass through o. He has confused the point of the "divide"—which is, naturally, the point of *greatest pressure*—with the *centre of pressure*. When the plate is completely immersed the two are the same; but when the immersion is only partial, then, as was pointed out (later) by Colonel Duchemin in *Les lois de la résistance des fluides*, the "divide" is at the centre of figure—although the centre of pressure is to be found above this point. It is clear that if the point of greatest pressure is at o, in consequence of what Avanzini calls the "lip" (labbra), the centre of pressure must be above this.

In the diagram (Fig. 3), let A B represent a vertical section of the plate and C D the surface of the water, o being the centre of figure of the immersed part of the plate. If then the shaded part be taken to express the pressure on the plate, the *maximum* pressure will be at o Y, whilst the centre of pressure will be at some point e, above this; the position of which will depend on the amount of pressure on the plate above C D.

Fig. 2 is Avanzini's diagram of the plate when the angle of attack is acute. In this case, also, the centre of pressure is taken as the divide, which certainly appears incorrect. I am unable to say whether when the angle is acute the division follows the same law as it does when the plate is wholly immersed. Duchemin made no experiments to determine this, and I am unaware that anyone else has ever taken up the subject since. If only someone would take up the work for water which Eiffel and the Duc de Guiche have undertaken for air, i.e. *map out the contours of pressure* on plates moving in water, much information about the movement of water might be

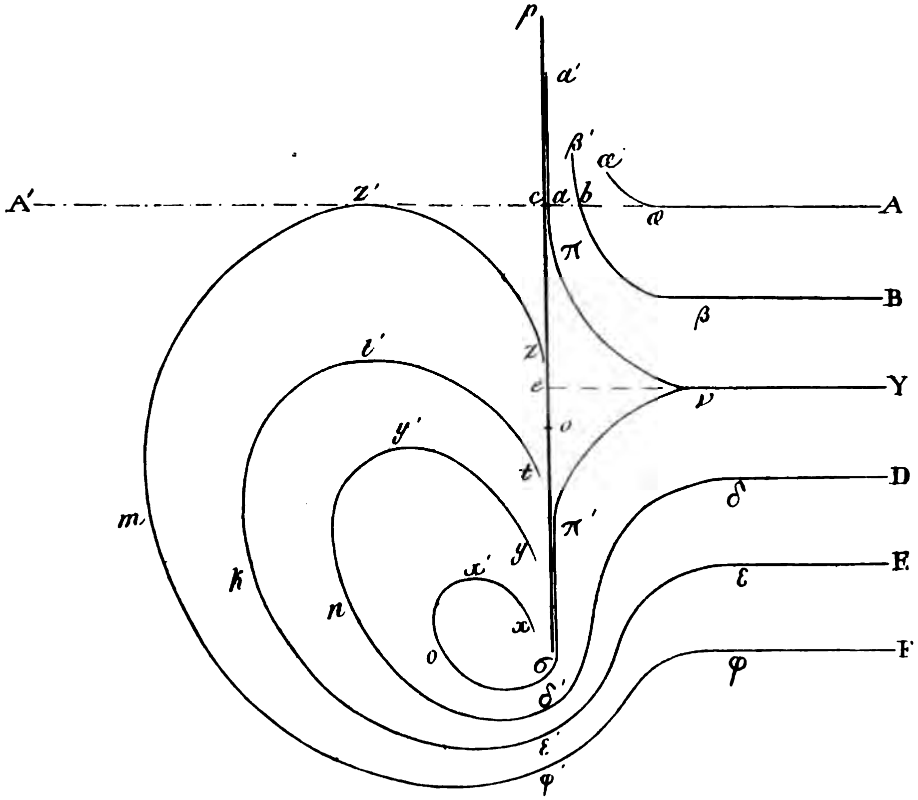
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Fig. 1.

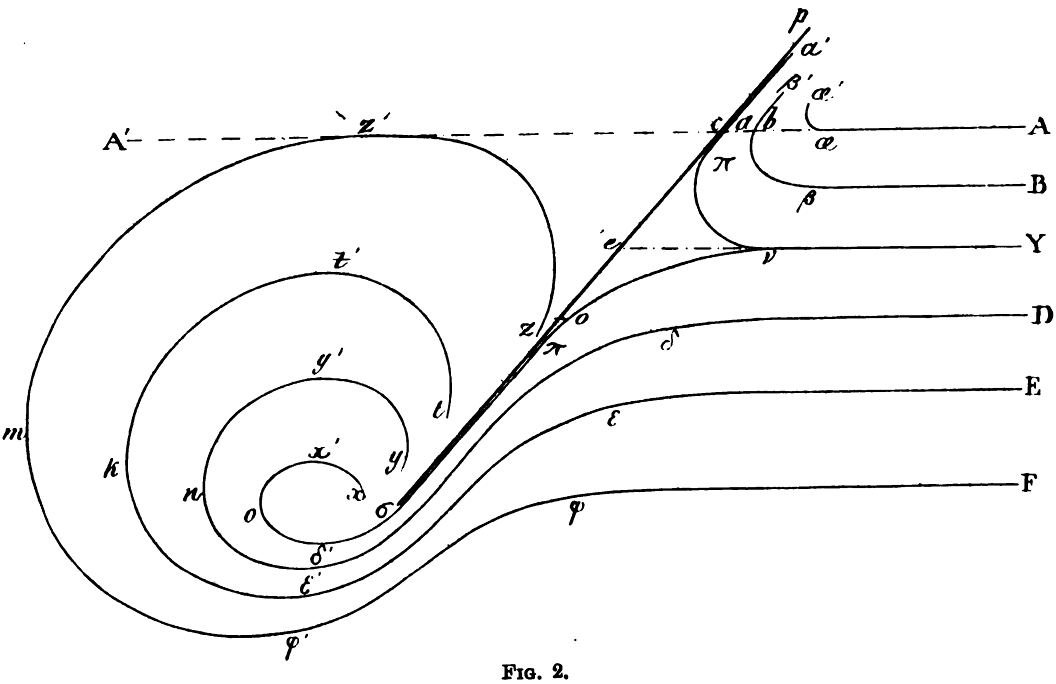
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Fig. 2.

gained. Dr. Ahlborn has arrived at the conclusion that there can be no doubt as to the complete similarity of the flow in air and in water. That this is, generally, true I think cannot be doubted, but the "similarity" must not be pressed too closely. Riabouchinsky's beautiful photographs of "spectres" show what, I cannot help thinking, are very distinct signs of compression in the air; it would appear as if there were a vibrating "cushion" of air between the plates, or other bodies, and the stream lines of the air—the white powder being visible at the "nodes," where the air is at rest. This cushion of vibrating air would have a tendency to blur some of the contours of pressure,

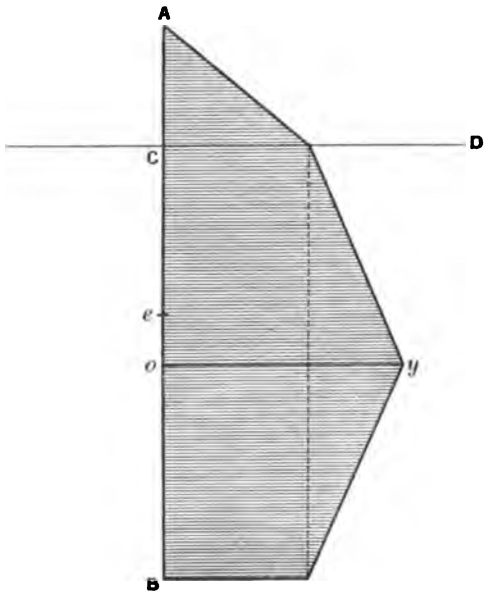

Fig. 3.

and so mask some of the effects caused by the differences of velocity of the various stream lines. To properly understand the motion of air, it is necessary to commence by studying the more simple fluid, water.

There are several other papers by Avanzini in the *Memorie* of the *Istituto Nazionale Italiano*, but they are chiefly attacks on the theory of liquid resistance of the famous Spanish Admiral, Don George Juan,* and so have no present interest.

* Libraries of British Museum and Royal Society: copies to be found in Spanish, Italian, and French.